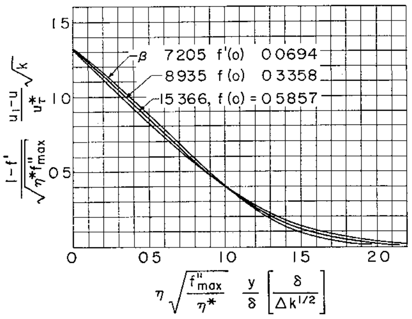


Fig 1 Computed profiles

of $U/U_1$ at the wall The second is a consequence of Eq (1) The third states that $U/U_1$ approaches unity asymptotically

Solutions to Eq (2) were obtained on an electronic computer for various combinations of $f'(0)$ (which allows for variation in the distance along the surface $x$) and $\beta$ (which allows for variation in the transpiration velocity $V_0$) Profiles of $U/U_1$ as a function of $\eta$ were used to construct defect plots of the form $(U - U_1)/U_\tau^*$ vs $y/\delta$, where $U_\tau^*$ is a friction velocity based on the *maximum* computed shear stress From Clauser's expression for the eddy viscosity,

$$\epsilon = k\rho U_1\delta^* \tag{5}$$

it is possible to show that

$$U_\tau^* = (\tau^*/\rho)^{1/2} = (kU_1{}^2\eta^* f_{\max}'')^{1/2} \tag{6}$$

where

$$\eta^* = \int_0^\infty \left(1 - \frac{U}{U_1}\right) d\eta$$

Thus,

$$\frac{U_1 - U}{U_\tau^*} = \frac{1 - f'}{(k\eta^* f_{\max}'')^{1/2}} \tag{7}$$

The coordinate $\eta$ may be transformed into $y/\delta$ with the aid of a relationship given by Clauser:

$$\frac{y}{\delta} = \frac{\Delta}{\delta}\left(\frac{\Delta\, kf_{\max}''}{\eta^*}\right)^{1/2}\eta \tag{8}$$

where

$$\frac{\Delta}{\delta} = \int_0^1 \frac{U_1 - U}{U_\tau^*} d\left(\frac{y}{\delta}\right)$$

The transformed coordinates contain the unknown quantities $k$ and $\Delta/\delta$, which may be determined only by experiment However, if an equilibrium layer is, in fact, to exist, these quantities must be constant for a given equilibrium profile and may therefore be deleted for the purpose of determining the existence of such profiles

Figure 1 presents some of the results in terms of these modified coordinates It is apparent that all three of these curves

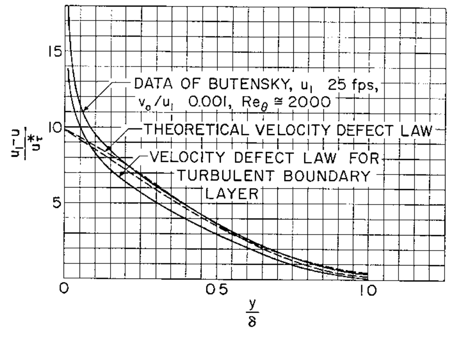


Fig 2 Comparison with experimental data

could be satisfactorily approximated by a single universal profile Additional results show this to be true for a very wide range of $\beta$ and $f'(0)$ Moreover, the effect of $\beta$ is found to be very weak, the spread between the curves being associated almost entirely with the changes in $f'(0)$ Thus, in terms of the modified, transformed coordinates, nearly all solutions lead to a single defect plot It is therefore not possible, on analytical grounds, to select a similarity parameter analogous to Clauser's $\delta^* dp/\tau_0 dx$ Significantly, the use of other scale velocities, such as a friction velocity based on the wall shear-stress, did not lead to a universal profile

In the authors' judgment, Butensky's data[3] provide the best shear-stress profiles presently available for a transpired turbulent boundary layer Unfortunately, his experimental conditions did not yield a universal profile Nevertheless, if one sets $k = 0\,018$ as found by Clauser and uses the value of $\Delta/\delta$ associated with a particular profile, the analytical prediction of that experimental profile is quite satisfactory, as illustrated in Fig 2 The defect law for a nontranspired boundary layer is shown for comparison, and the dashed lines indicate the spread of the theoretical results

It remains for experiment to determine what condition leads to an equilibrium profile, i e , constant $\Delta/\delta$ This work is now underway in the authors' laboratory

## References

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# Elastic Stability of Castellated Plates

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In the design of aircraft wing ribs or fuselage frames, it is often necessary to connect the various structural elements by a plate containing numerous cutouts for stringers and longerons (Fig 1) Such castellated plates are loaded in shear and usually fall because of buckling of the free edge

A data sheet[1] issued by the Royal Aeronautical Society in England enables the calculation of the allowable load on castellation teeth of various geometric proportions The information appearing on this data sheet summarizes the results of a large number of tests The purpose of this note is to show that the shear stress corresponding to initial buckling of the teeth can readily be predicted theoretically and that it is in good agreement with experimental data

We describe the lateral deflection of a castellation by the expression

$$w = A \sin(\pi x/h) \sin(\pi y/2b) \tag{1}$$

The shape represented by Eq (1) is in close agreement with experimental observations The value of $w$ is zero along three sides consisting of the tension edge ($y = 0$) and the root and skin lines ($x = 0$ and $x = h$) It is a maximum at the midpoint of the compression edge

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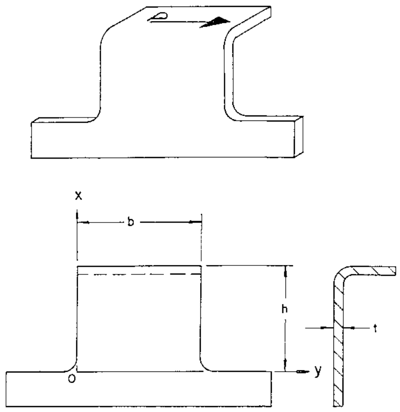


**Fig 1 Cooidinate system and typical castellation**

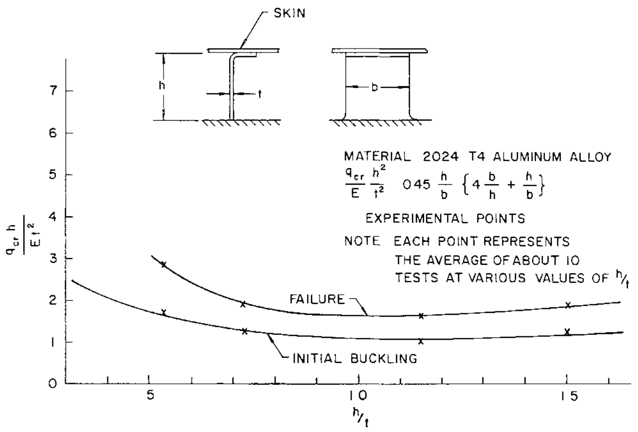


**Fig 2 Compaiison of theoretical and experimental iesults**

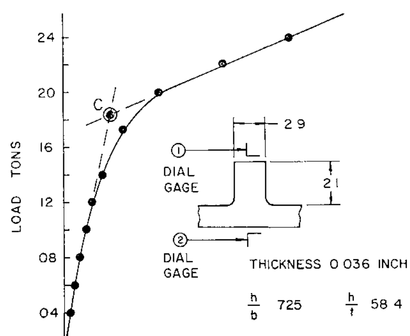


**Fig 3 Typical load-deflection curves from Ref 2**

The strain energy and the work done by the external forces now follow from the equations

$$V = \frac{1D}{2}\int_0^b\int_0^h\left\{\left[\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}\right]^2 - 2(1-\nu)\left[\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2}-\left(\frac{\partial^2 w}{\partial x \partial y}\right)^2\right]\right\} dx\, dy \quad (2)$$

and

$$T = \frac{1}{2}\int_0^b\int_0^h\left\{N_x\left(\frac{\partial w}{\partial x}\right)^2 + 2N_{xy}\frac{\partial w}{\partial x}\frac{\partial w}{\partial y}\right\} dx\, dy \quad (3)$$

where $D = Et^3/12(1-\nu^2)$, and all other symbols have their customary meaning In Eq (3) we require some knowledge of the direct and shear forces, $N_x$ and $N_{xy}$, respectively A reasonable distribution is given by

$$N_x = (Pt/I)(xy - \tfrac{1}{2}bx) \quad (4)$$

and

$$N_{xy} = \tfrac{3}{2}(P/b^3)(b^2 - y^2) \quad (5)$$

Upon su bstitution, we obtain the critical shear stress $q_c$ :

$$\frac{q_{cr}h^2}{Et^2} = 0\ 045\,\frac{h}{b}\left\{4\,\frac{b}{h}+\frac{h}{b}\right\}^2 \quad (6)$$

This relationship is compared in Fig 2 with test results obtained from Ref 2 A typical load-deflection curve is shown in Fig 3 The point of intersection $C$ of the initial and final slopes of the load deflection curve is assumed to determine the critical buckling load and is chosen for comparison with theoretical results Clearly, our analysis is restricted to stress levels at which plasticity does not play a dominant role

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# Calculation of Cylindrical Blast Wave Propagation with Counterpressure

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### Nomenclatuie

$a_\infty$ = speed of sound in undisturbed gas
$C_D$ = drag coefficient
$d$ = diameter of cylinder
$E$ = explosion energy per unit length of charge or drag of equivalent body in hypersonic flight
$m$ = Lagrangian variable, $\rho_\infty R^2/2$
$M$ = Mach number, $u_\infty/a_\infty$
$p_\infty$ = pressure of undisturbed gas
$p_s$ = pressure behind shock wave
$p_p$ = pressure at piston surface
$q$ = $a_\infty{}^2/\dot{R}_0{}^2$
$R$ = shock-wave coordinate at time when gas particle crosses shock wave
$R_0$ = shock-wave coordinate at any time
$R_p$ = coordinate of piston surface at any time
$t$ = time
$u_\infty$ = velocity of undisturbed gas with respect to observer fixed in body frame in equivalent hypersonic flight problem
$x$ = axial distance in equivalent hypersonic flow problem
$\gamma$ = ratio of specific heats
$\epsilon$ = $(\gamma-1)/(\gamma+1)$
$\rho_\infty$ = density of undisturbed gas
$\rho_s$ = density behind shock wave

We propose to show how the integral method of Chernyi[1] may be used to furnish a second approximation to the shock-wave shape and pressure predicted by cylindrical blast-wave theory [2] The comparative ease with which the result is obtained, and the good agreement with experiment, affords an excellent illustration of the potential value of this powerful yet simple method in the analysis of hypersonic flows

The starting point in our analysis is the simplified version of the energy-integral equation for the problem of a violent explosion due to a line charge, followed by the expansion of a

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